

Lesson 6, Part 2:  
Linear Mixed Effects Models

# **This Lesson's Goals**

Learn about other methods for LMEMs

Update our LMEMs in R

Summarise results in an R Markdown document

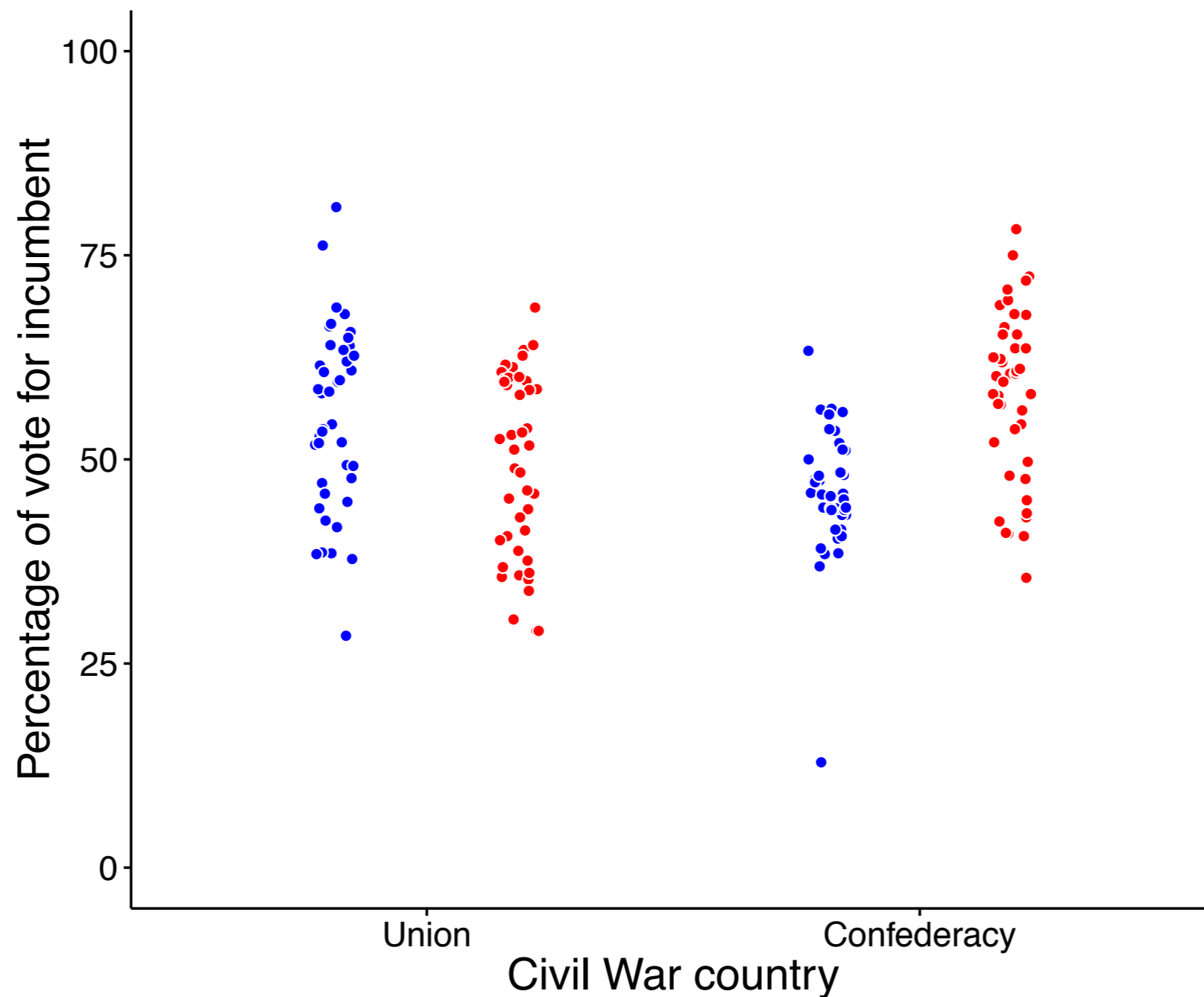
But, in the ANOVA we got rid of our baseline issue, can I do that with an LMEM?

# **Math (Part 1)**

$$y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{1i}X_{2i} + e_i$$

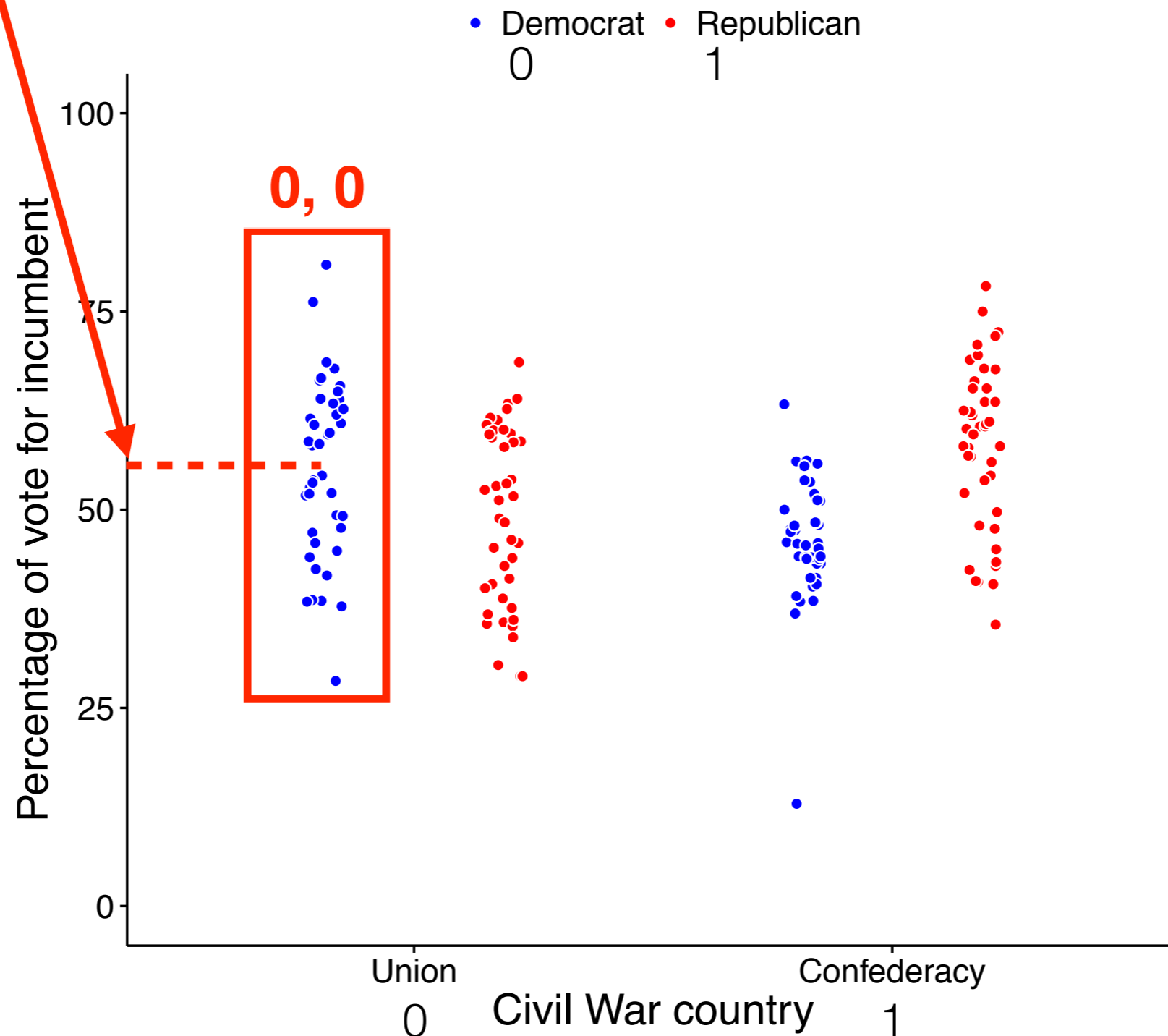
Percentage of Votes for Incumbent  
by Country in Civil War and Party of Incumbent

• Democrat • Republican



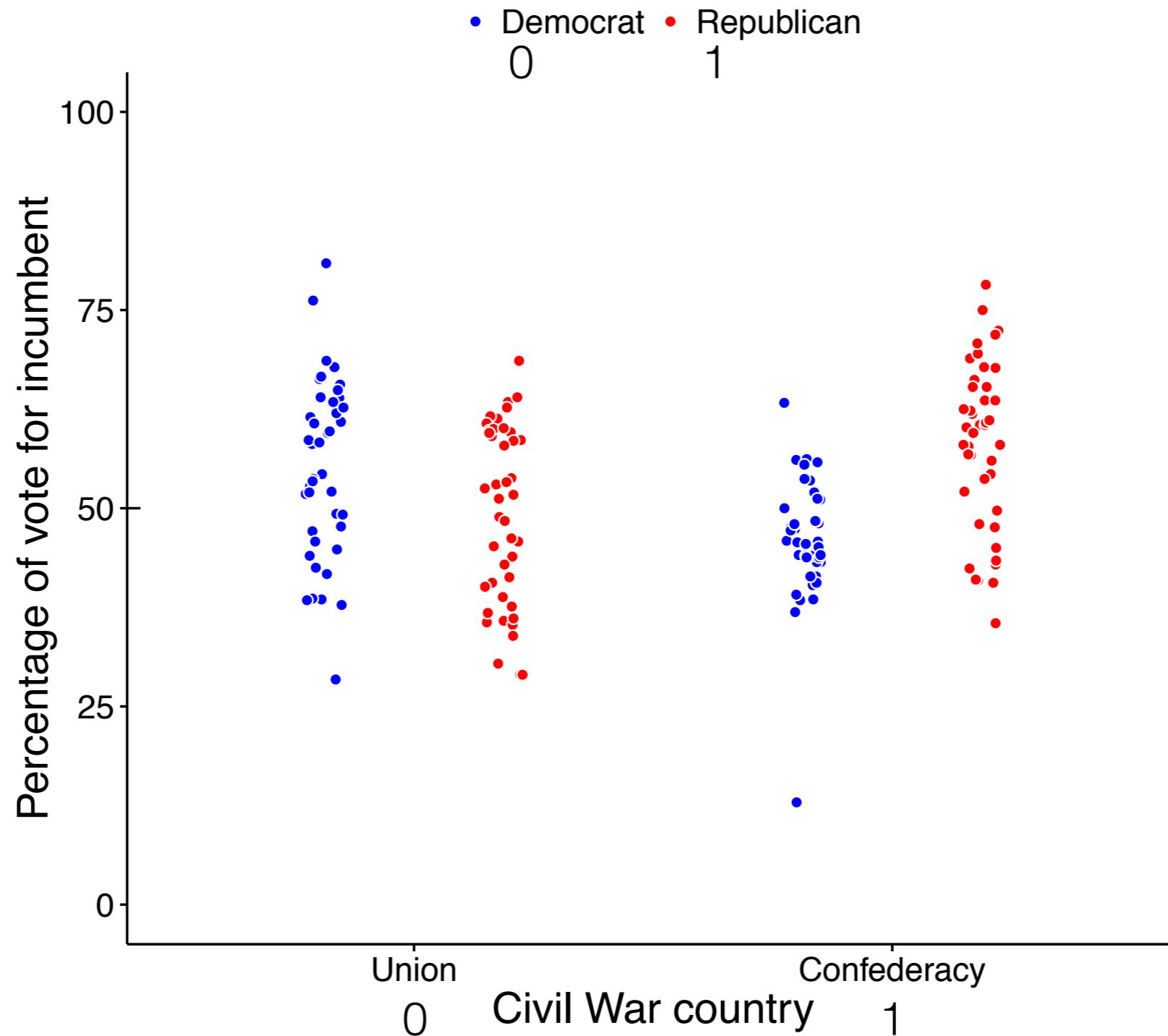
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Percentage of Votes for Incumbent  
by Country in Civil War and Party of Incumbent



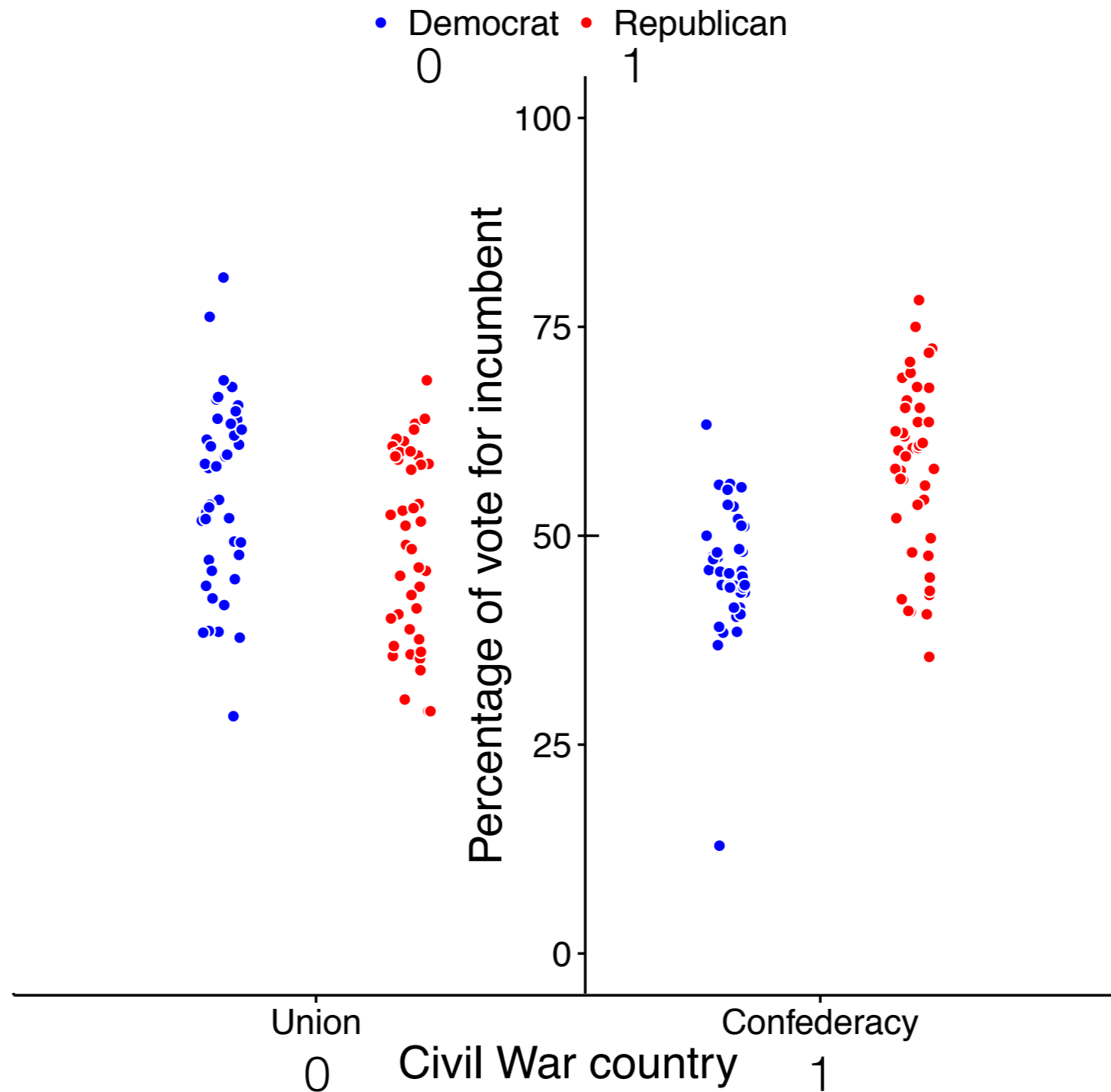
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Percentage of Votes for Incumbent  
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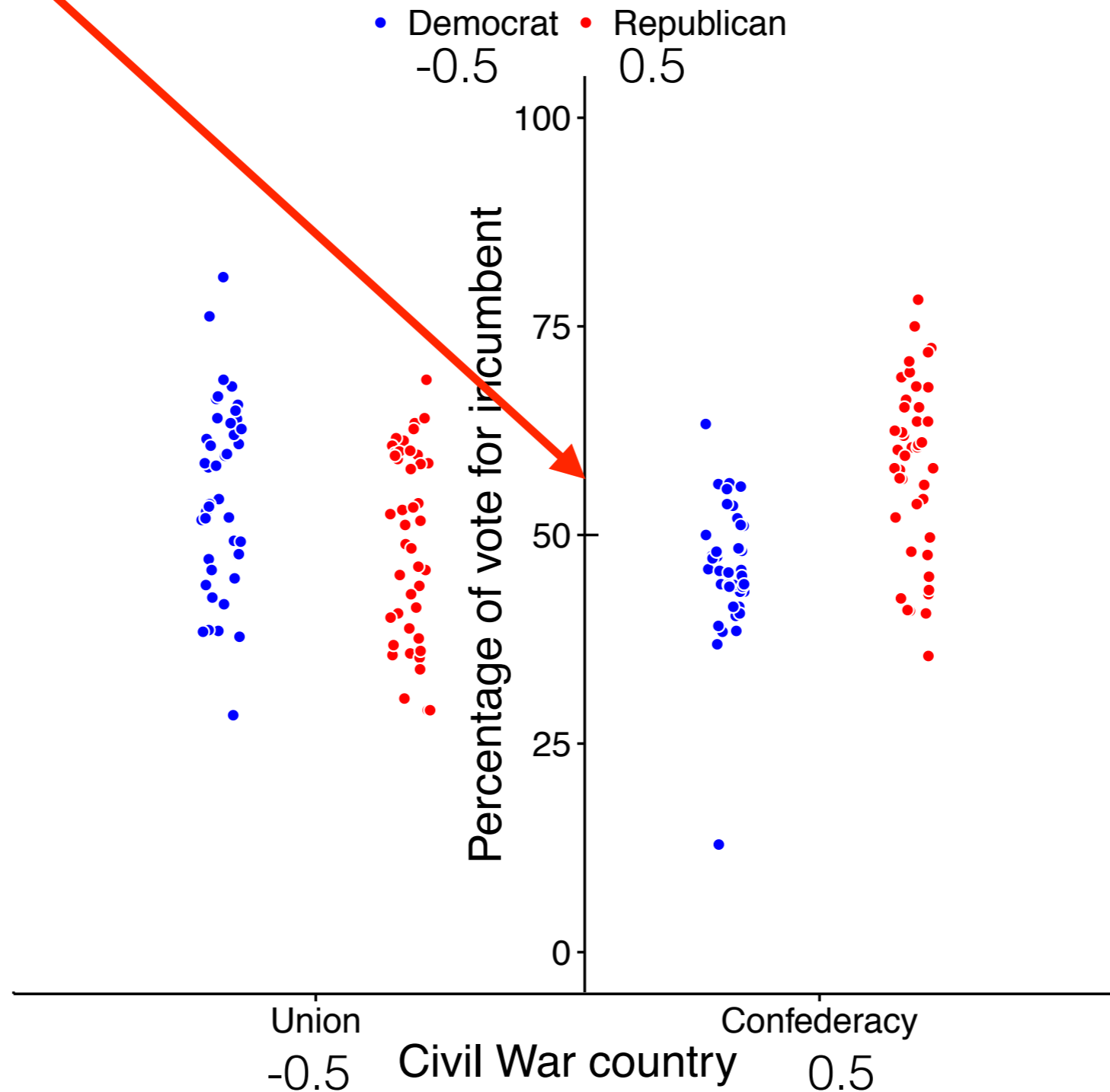
Percentage of Votes for Incumbent  
by Country in Civil War and Party of Incumbent





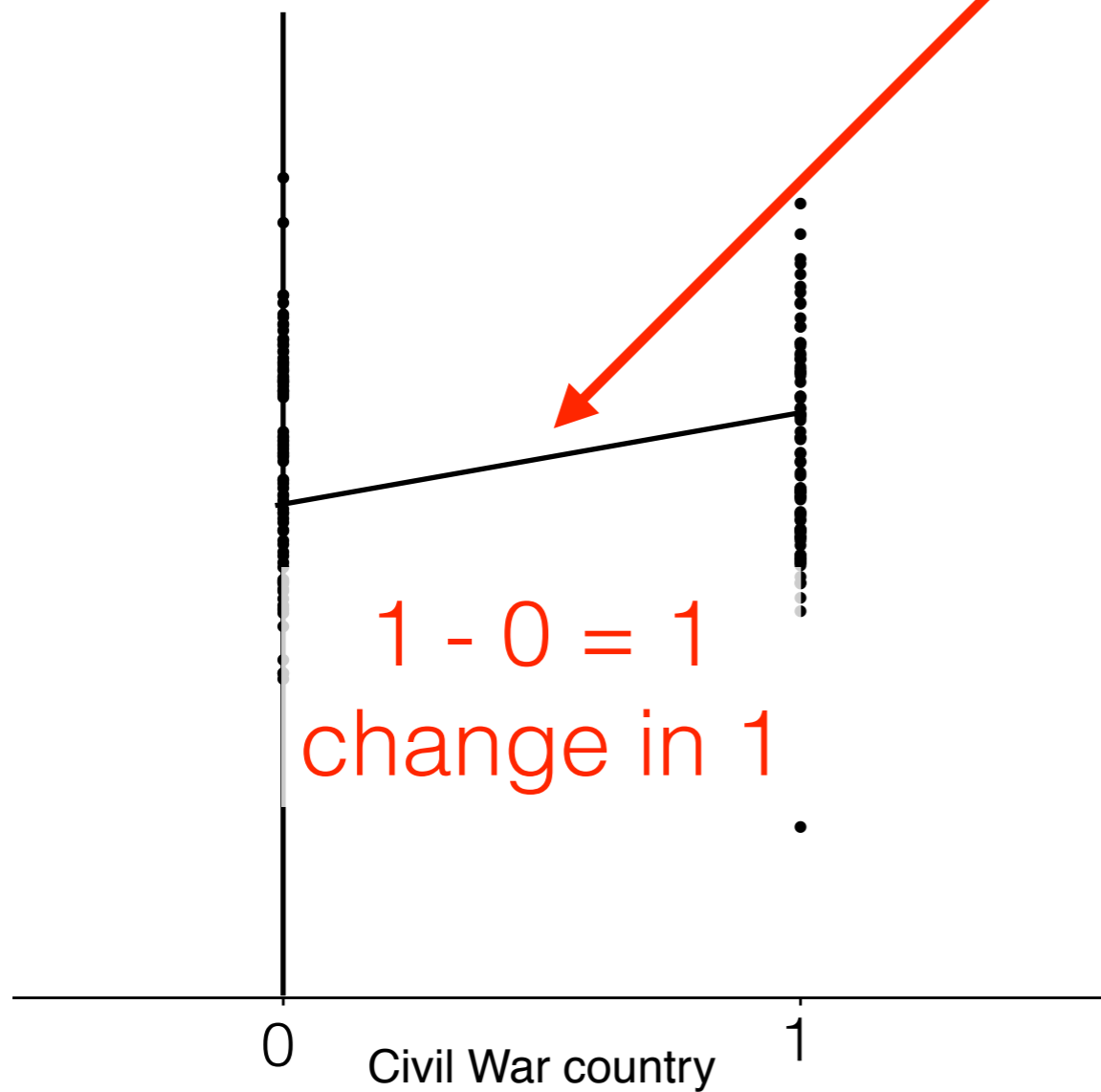
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Percentage of Votes for Incumbent  
by Country in Civil War and Party of Incumbent

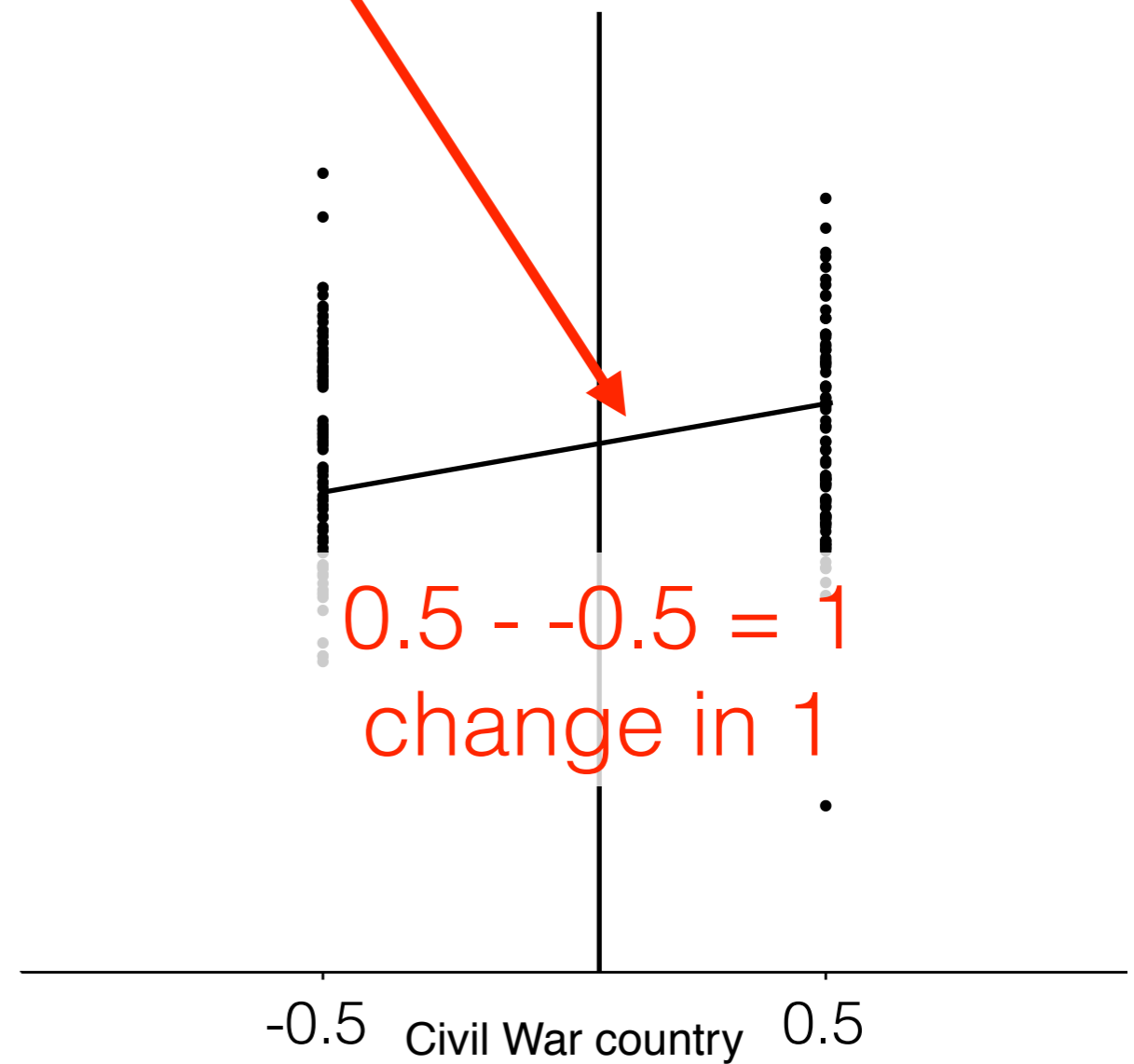


$$y_i = a + bx_i$$

**dummy coding**

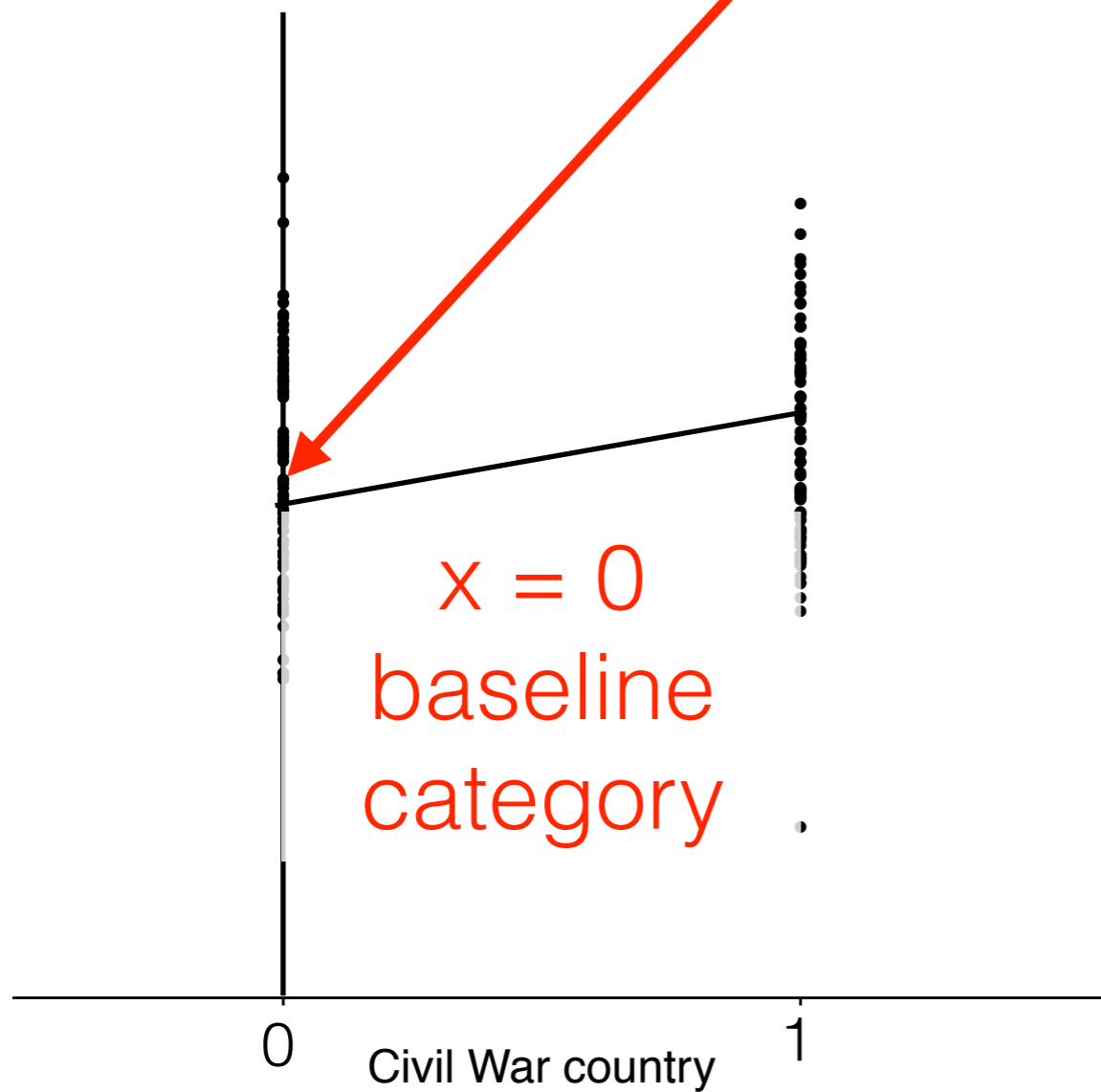


**contrast coding**

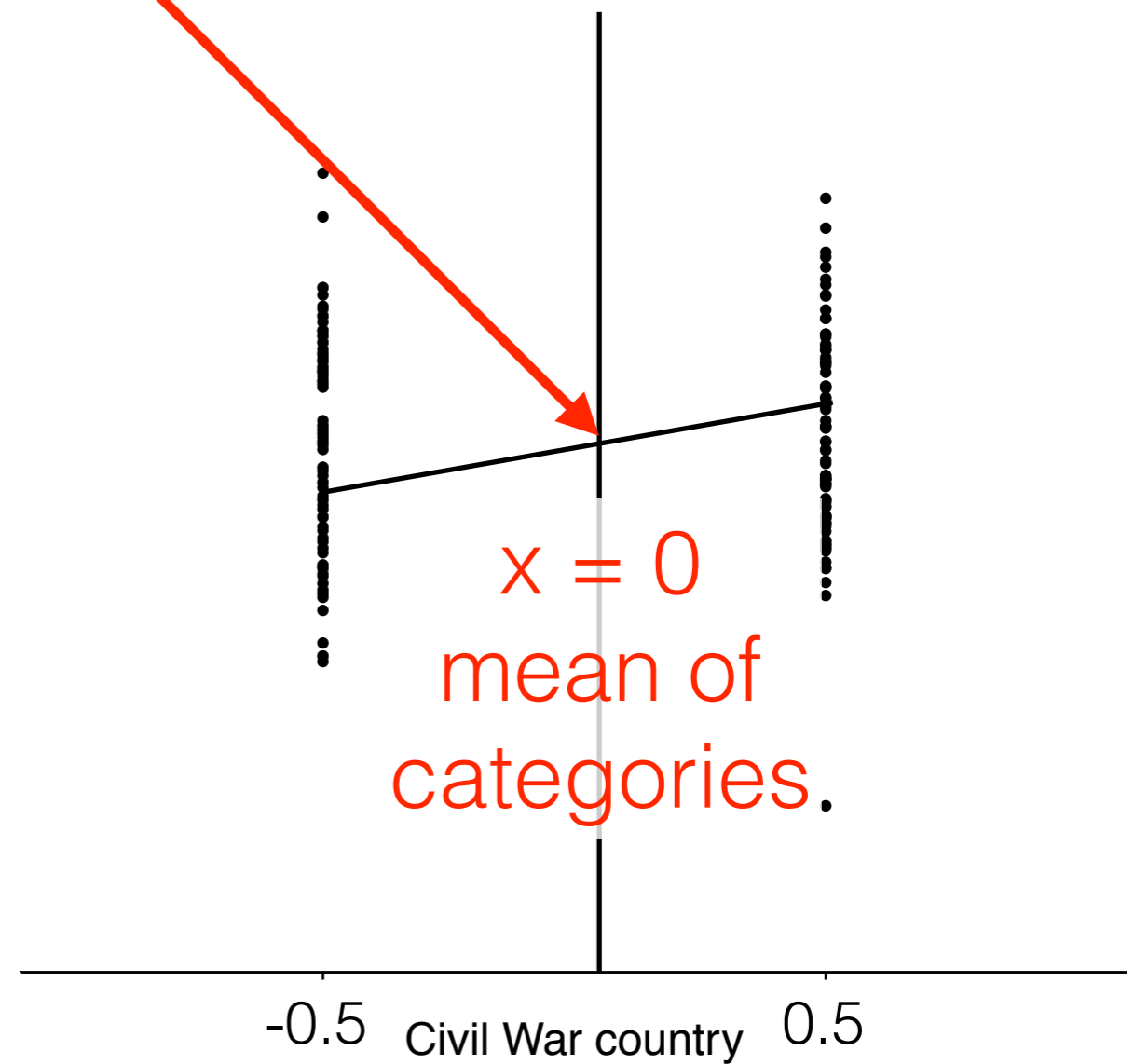


$$y_i = a + bx_i$$

**dummy coding**

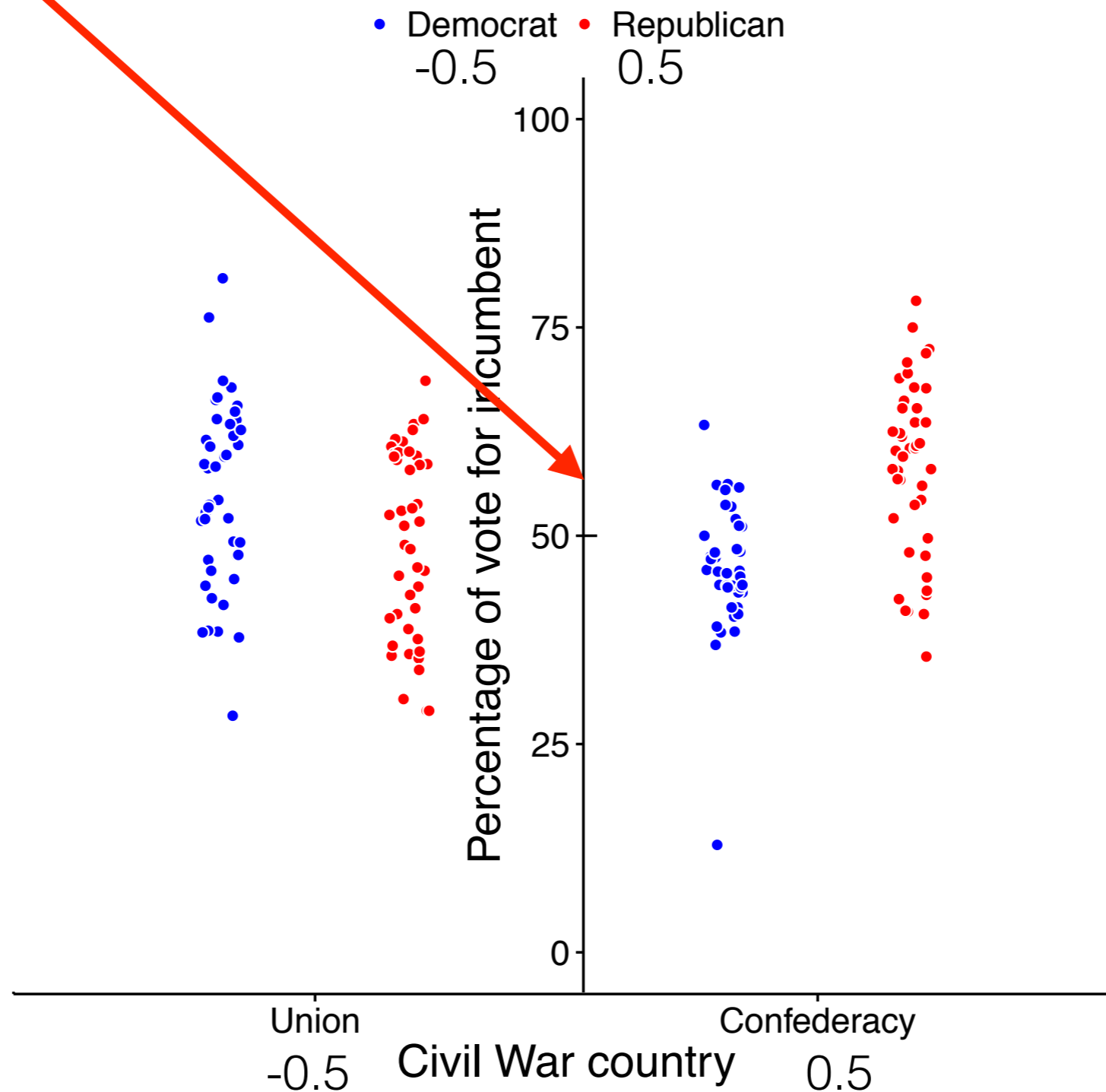


**contrast coding**



$$y_i = a + b_1X_{1i} + b_2X_{2i} + b_3X_{1i}X_{2i} + e_i$$

Percentage of Votes for Incumbent  
by Country in Civil War and Party of Incumbent



# **R Code (Part 1)**

## lme4

$$y_i = a + a_s + a_y + (b_{s1} + b_1)x_{1i} + (b_{y1} + b_2)x_{2i} + b_3x_{1i}x_{2i} + e_i$$

```
lmer(perc_votes_incumbent ~  
incumbent_party_contrast * civil_war_contrast  
+ (1+incumbent_party_contrast|state)  
+ (1+civil_war_contrast|year))
```

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	52.04864	3.11346	16.717
incumbent_party_contrast	2.92546	6.37541	0.459
civil_war_contrast	0.04273	2.77575	0.015
incumbent_party_contrast:civil_war_contrast	18.39637	6.18901	2.972

## LMEM with dummy coding

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	55.164	5.591	9.866
incumbent_partyRepublican	-6.273	8.012	-0.783
civil_warConfederacy	-9.155	4.183	-2.189
incumbent_partyRepublican:civil_warConfederacy	18.396	6.189	2.972

## LMEM with contrast coding

Fixed effects:

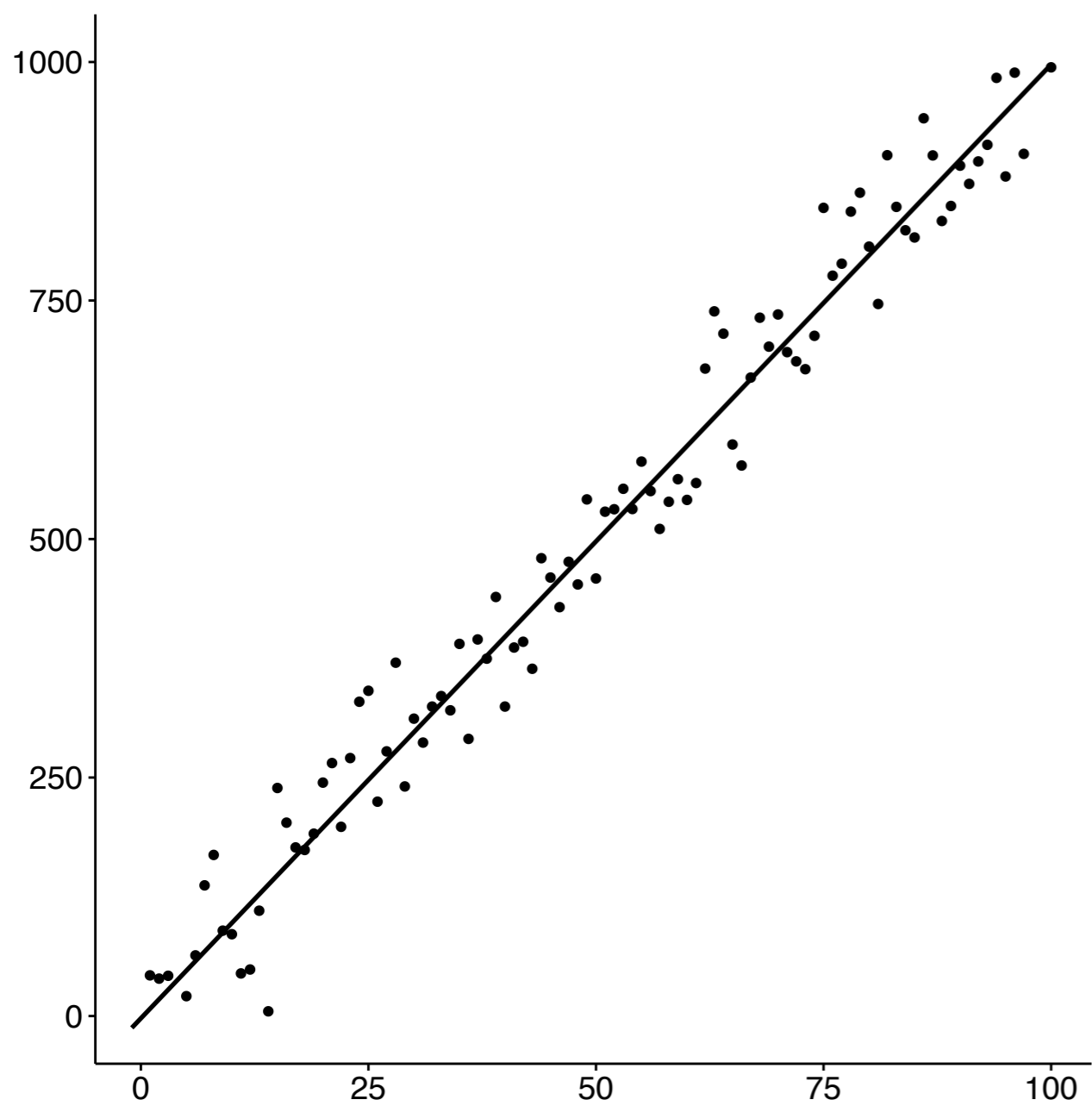
	Estimate	Std. Error	t value
(Intercept)	52.04864	3.11346	16.717
incumbent_party_contrast	2.92546	6.37541	0.459
civil_war_contrast	0.04273	2.77575	0.015
incumbent_party_contrast:civil_war_contrast	18.39637	6.18901	2.972

How do I get p-values out of this?

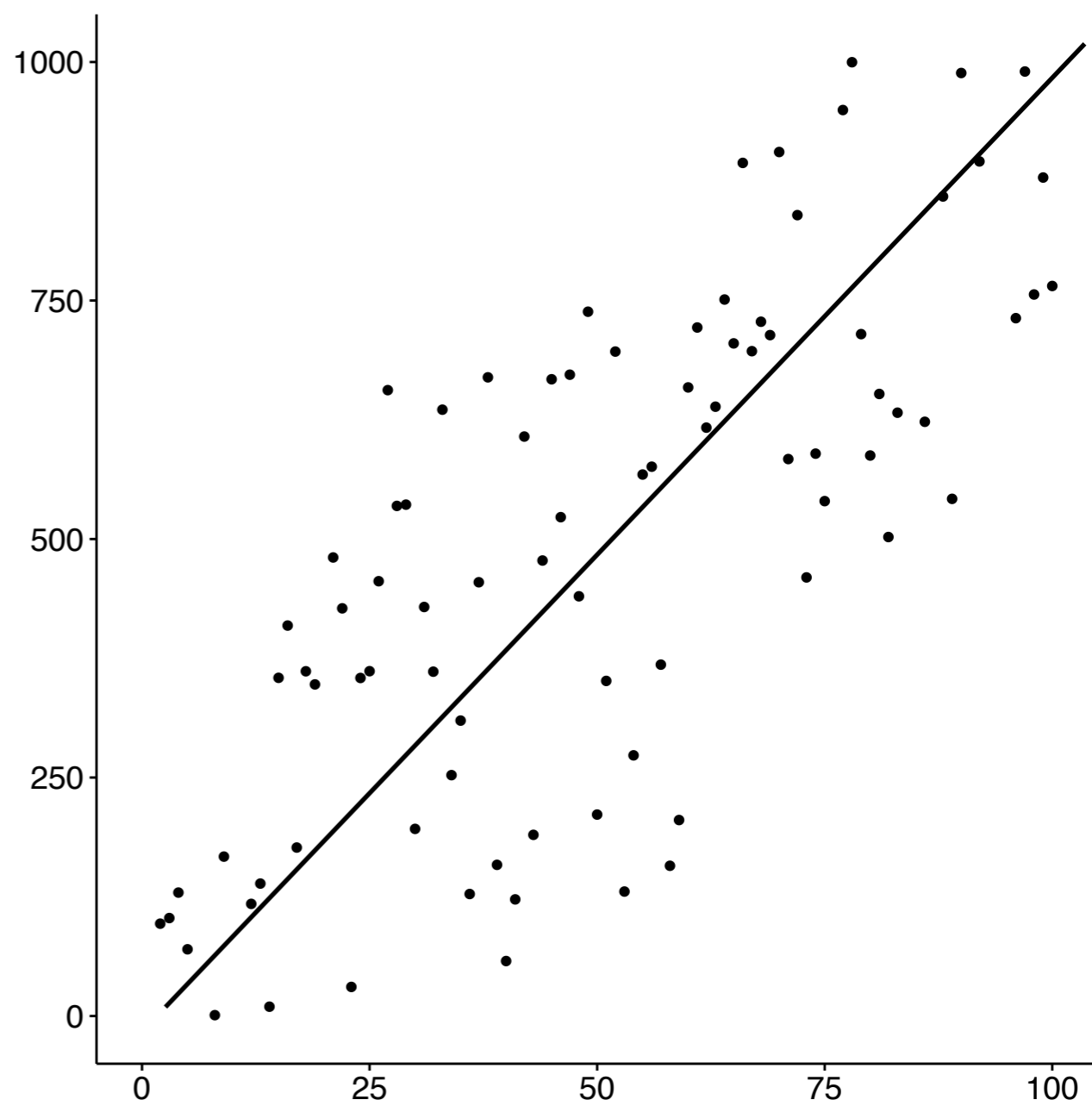


# **Math (Part 2)**

# very good fit



# not as good fit



# **R Code (Part 2)**

$$y_i = a + a_s + a_y + (b_{s1} + b_1)x_{1i} + (b_{y1} + b_2)x_{2i} + b_3x_{1i}x_{2i} + e_i$$

```
lmer(perc_votes_incumbent ~  
incumbent_party_contrast * civil_war_contrast  
+ (1+incumbent_party_contrast|state)  
+ (1+civil_war_contrast|year), REML = F)
```

$$y_i = a + a_s + a_y + (b_{s1} + b_1)x_{1i} + (b_{y1} + b_2)x_{2i} + b_3x_{1i}x_{2i} - b_1x_{1i} + e_i$$

```
lmer(perc_votes_incumbent ~  
incumbent_party_contrast * civil_war_contrast  
- incumbent_party_contrast  
+ (1+incumbent_party_contrast|state)  
+ (1+civil_war_contrast|year), REML = F)
```

$$y_i = a + a_s + a_y + (b_{s1} + b_1)x_{1i} + (b_{y1} + b_2)x_{2i} + b_3x_{1i}x_{2i} + e_i$$

$$y_i = a + a_s + a_y + (b_{s1} + b_1)x_{1i} + (b_{y1} + b_2)x_{2i} + b_3x_{1i}x_{2i} - b_1x_{1i} + e_i$$

`anova(m1, m2_party)`

	Df	AIC	BIC	logLik	deviance	Chisq	Chi	Df	Pr(>Chisq)
m2_party	10	1153.5	1185.1	-566.74	1133.5				
m1	11	1155.2	1190.0	-566.61	1133.2	0.2706		1	0.6029

smaller AIC  
means better fit

not significant  
at  $p < 0.05$

# LMEM summary

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	52.04934	2.70659	19.231
incumbent_party_contrast	2.92405	5.57867	0.524
civil_war_contrast	0.04413	2.43834	0.018
incumbent_party_contrast:civil_war_contrast	18.39355	5.57298	3.300

## model comparison for incumbent party

	Df	AIC	BIC	logLik	deviance	Chisq	Chi	Df	Pr(>Chisq)
m2_party	10	1153.5	1185.1	-566.74	1133.5				
m1	11	1155.2	1190.0	-566.61	1133.2	0.2706		1	0.6029

## model comparison for civil war

	Df	AIC	BIC	logLik	deviance	Chisq	Chi	Df	Pr(>Chisq)
m3_country	10	1153.2	1184.9	-566.61	1133.2				
m1	11	1155.2	1190.0	-566.61	1133.2	3e-04		1	0.9856

## model comparison for incumbent party x civil war

	Df	AIC	BIC	logLik	deviance	Chisq	Chi	Df	Pr(>Chisq)
m4_partyxcountry	10	1160.8	1192.4	-570.39	1140.8				
m1	11	1155.2	1190.0	-566.61	1133.2	7.567		1	0.005945

**Lab**

# Data set: Stroop Task

Congruency: Are responses to incongruent trials less accurate and slower than to congruent trials?

Experiment half: Are responses more accurate and faster in the second half of the experiment than the first half of the experiment?

Congruency x Experiment half: Is there an interaction between these variables?

## **accuracy (logistic)**

logit  $p_i$  = accuracy  
x1 = congruency  
x2 = experiment half  
r1 = participant  
r2 = item

## **reaction times (linear)**

$y_i$  = reaction times  
x1 = congruency  
x2 = experiment half  
r1 = participant  
r2 = item

*source: real students!*



**dplyr**

```
data_accuracy_stats = data_accuracy_clean
```

# dplyr

```
data_accuracy_stats = data_accuracy_clean %>%  
  mutate(congruency_contrast =  
  )
```

make new  
variable



# dplyr

```
data_accuracy_stats = data_accuracy_clean %>%  
  mutate(congruency_contrast =  
    ifelse(  
      ) )
```

make new  
variable



make condition  
statement



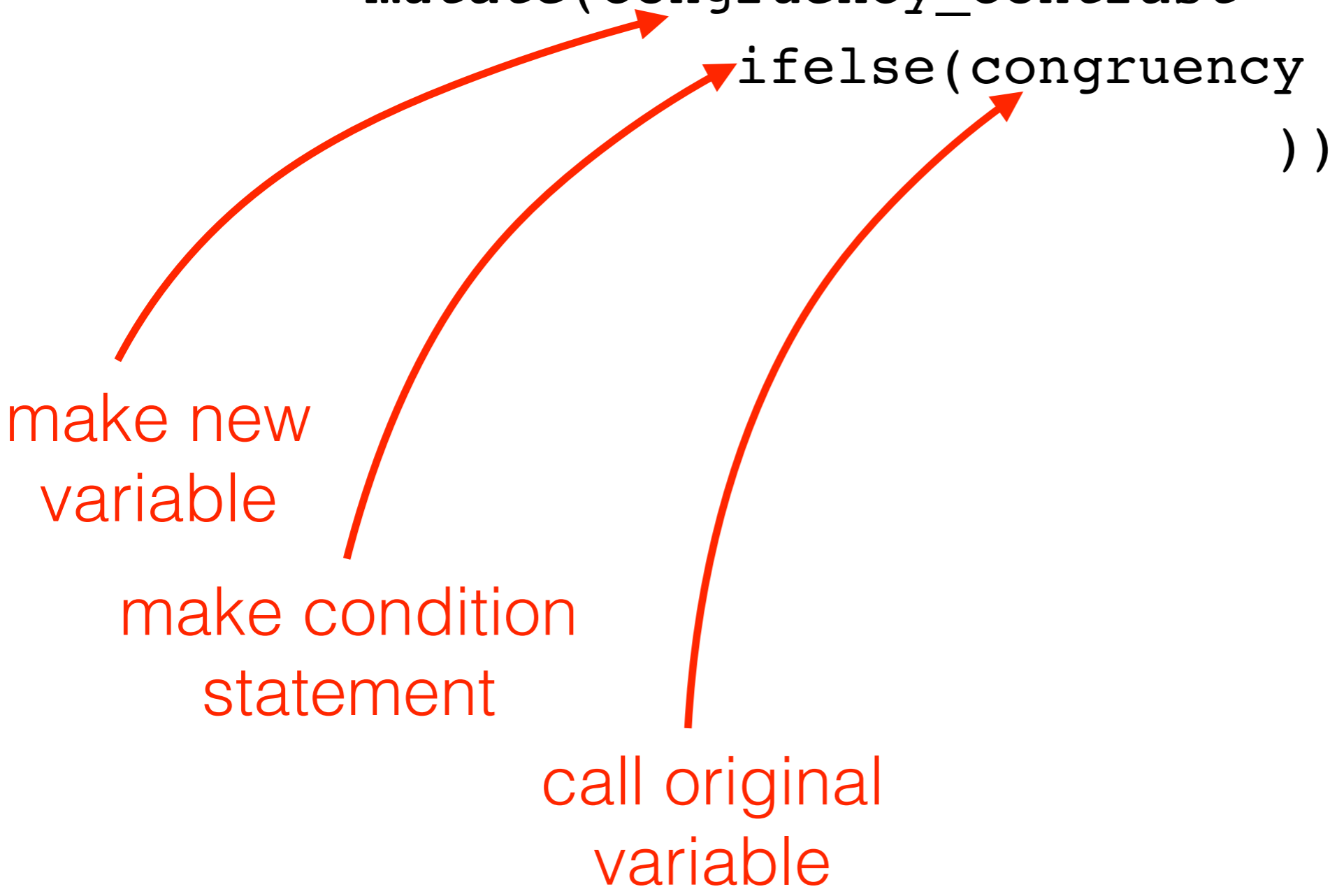
# dplyr

```
data_accuracy_stats = data_accuracy_clean %>%  
  mutate(congruency_contrast =  
    ifelse(congruency  
           ))
```

make new  
variable

make condition  
statement

call original  
variable



# dplyr

```
data_accuracy_stats = data_accuracy_clean %>%  
  mutate(congruency_contrast =  
    ifelse(congruency == "con"  
           ))
```

make new  
variable

make condition  
statement

call original  
variable

reference  
level

# dplyr

```
data_accuracy_stats = data_accuracy_clean %>%  
  mutate(congruency_contrast =  
    ifelse(congruency == "con",  
           -0.5,  
           ))
```

make new  
variable

make condition  
statement

call original  
variable

reference  
level

new value  
of con

# dplyr

```
data_accuracy_stats = data_accuracy_clean %>%
```

```
  mutate(congruency_contrast =
```

```
    ifelse(congruency == "con",  
          -0.5, 0.5))
```

make new  
variable

make condition  
statement

call original  
variable

reference  
level

new value  
of con

new value  
of incon